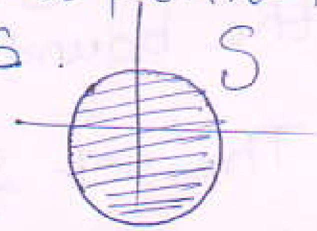
- Upper Half plane (UHP) : $\{z = x+iy \mid y>0\}$
- Lower Half plane (LHP) : $\{z = x+iy \mid y<0\}$
- Right Half plane (RHP) : $\{z = x+iy \mid x>0\}$
- Left Half Plane (LHP) : $\{z = x+iy \mid x<0\}$.
- Set S is <u>open</u> if every point of S has a neighborhood (nbhd) consisting entirely of points belonging to S. Such a point is called an <u>interior point of S</u>.
  — Is this an open set ?

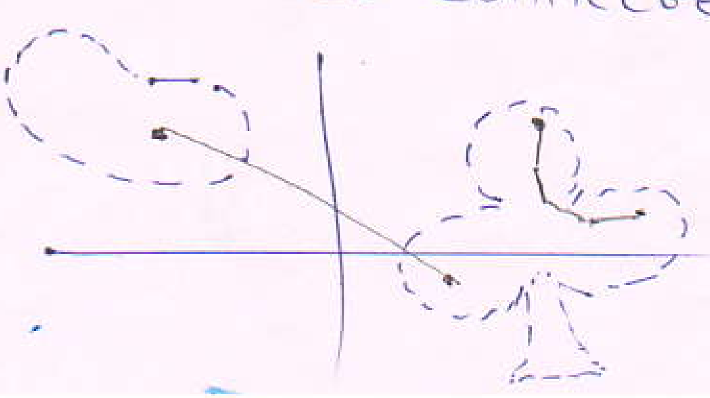


  — What about the following ?

- Similarly define exterior point of S, boundary point of S.

- An ~~open~~ set S is <u>connected</u> if any two of its points can be joined by a broken line of finitely many straight-line segment all of whose points belong to S.

  — Is this set connected ?



- A set is <u>open</u> if it contains none of its boundary points.
- A set is <u>closed</u> if it contains all of its boundary points.
- The <u>closure</u> of a set S is the closed set consisting of all points in S together with the boundary points of S.
- The set $\{z \in \mathbb{C} : 0 < |z| \leq 1\}$ (punctured disk) is neither open nor closed.

- A set is <u>bounded</u> if every point of S lies inside some circle $|z| = R$, otherwise it is <u>unbounded</u>.

- A point z_0 is said to be an <u>accumulation point</u> of a set S if each deleted neighborhood of z_0 contains at least one point of S.

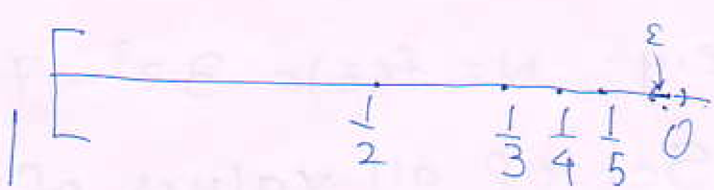
Let $S = \{z \in \mathbb{C} : |z| < 1\}$ & let z_0 be a boundary point of S.

Then z_0 is an accumulation point of S.

- What about the interior points of S?

- A closed set contains each of its accumulation points.

$$A = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$$



- Complement of a set S is the set of all points that do not belong to S.
- A set S is closed if its complement is open,

$|z| \leq 1$ is closed.

- A boundary point of a set S is a point every neighborhood of which contains both points that belong to S and points that do not belong to S.
- An open connected set is called a domain. A domain together with some, none, or all of its boundary points is region.

Complex function

Let $S \subset \mathbb{C}$.

A function $f: S \to \mathbb{C}$ is a rule that assigns to every z in S a complex number w. Thus,

$$w = f(z).$$

(S is the domain of ~~complex variable z~~ definition of f).

e.g. $w = f(z) = 3z^2 - 7z + 9$,

(Set of all values of f is the

Let $w = u + iv$, $z = x + iy$ & $w = f(z)$, then

$$w = f(z) = u(x, y) + i\,v(x, y).$$

- If $z = re^{i\theta}$, then $f(z) = u(r, \theta) + i\,v(r, \theta)$.

Limits and Continuity

We say $\lim_{z \to z_0} f(z) = l$ if f is defined in a neighborhood of z_0 (except may be at z_0 itself) and if the values of f are "close" to l for all z "close" to z_0. Precisely,

given an $\varepsilon > 0$, $\exists\, \delta > 0 \ni \forall\, z \neq z_0$ in $|z - z_0| < \delta$, we have $|f(z) - l| < \varepsilon$.

- In case of limits of real variables, x can approach x_0 only from the left or from the right.
- In case of complex variables, z may approach z_0 from any direction.
- If a limit exists, it is unique.
- $f(z)$ is said to be continuous in a domain if it is continuous at each point of this domain, with continuity at $z = z_0$ ...

Derivative

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{f(z_0+\Delta z) - f(z_0)}{\Delta z},$$

provided the limit exists.

- f is then said to be differentiable at z_0.
- With $\Delta z = z - z_0$, we have

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}.$$

Eg. ① $f(z) = z^3$ is differentiable $\forall\, z \in \mathbb{C}$ since

$$f'(z) = \lim_{\Delta z \to 0} \frac{(z+\Delta z)^3 - z^3}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{\cancel{z^3} + 3z^2\Delta z + 3z(\Delta z)^2 + (\Delta z)^3 - \cancel{z^3}}{\Delta z}$$

$$= 3z^2.$$

② $\bar{z}$ is not differentiable: Let $z = x+iy$ & Let $f(z) = \bar{z} = x - iy$. Suppose we write $\Delta z = \Delta x + i\Delta y$.

$$\frac{f(z+\Delta z) - f(z)}{\Delta z} = \frac{\overline{(z+\Delta z)} - \bar{z}}{\Delta z} = \frac{\overline{\Delta z}}{\Delta z} = \frac{\Delta x - i\Delta y}{\Delta x + i\Delta y}$$