TUTORIAL 5: MA 633: PARTITION THEORY

1. Logarithmically differentiate both sides of the equation

$$\sum_{n=0}^{\infty} p(n)q^n = \frac{1}{(q;q)_\infty}$$

and then prove Euler's recurrence relation for $p(n), n > 1$, namely,

$$np(n) = \sum_{j=0}^{n-1} p(j)\sigma(n-j),$$

where

$$\sigma(n) = \sum_{d|n} d.$$

2. For $j, k, n, m \in \mathbb{Z}$, prove that

$$\sum_{j+k=n} \frac{(a)_j (a)_k}{(b)_j (b)_k} (-1)^k = \begin{cases} 0, & \text{if } n = 2m+1, \\ \frac{(q)_\infty (b/a)_\infty (-b)_\infty (-q/a)_\infty}{(-q)_\infty (-b/a)_\infty (b)_\infty (q/a)_\infty} \frac{(a^2;q^2)_m}{(b^2;q^2)_m}, & \text{if } n = 2m. \end{cases}$$

3. For non-negative integers j, k, ℓ, m, n and $\omega = e^{2\pi i/3}$, prove that

$$\sum_{j+k+\ell=n} \frac{(a)_j (a)_k (a)_\ell}{(q)_j (q)_k (q)_\ell} \omega^{k+2\ell} = \begin{cases} 0, & \text{if } 3 \nmid n, \\ \frac{(a^3;q^3)_m}{(q^3;q^3)_m}, & \text{if } n = 3m. \end{cases}$$