MA 201: Tutorial 1

1. Determine the modulus and the principle argument of the following complex numbers:
(a) $-\pi - \pi i$ (b) $(1+i)^{20}$

2. Find z if $|z+i| = |z|$ and $\arg\left(\frac{z+i}{z}\right) = \frac{\pi}{4}$.

3. Prove that $\left|\dfrac{z}{|z|} - 1\right| \leq |\text{Arg}z|$.

4. Let z_1 and z_2 be two complex numbers. Prove the parallelogram law

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2),$$

and explain its geometrical interpretation.

5. Let n be an integer. Show that

$$(\sin\theta + i\cos\theta)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right).$$