MA 201: Complex Analysis - Tutorial 4

1. Evaluate $\int_C |z|\, dz$, where C is the circle $|z - 1| = 1$ described in the counterclockwise direction.

2. Evaluate $\int_C f(z)\, dz$ by using the formula $\int_{z_0}^{z_1} f(z)\, dz = F(z_1) - F(z_0)$ and then compare the result by using the formula $\int_C f(z)\, dz = \int_a^b f(z(t))\dot{z}(t)\, dt$, where:

   (a) $f(z) = z^4$ and C is the semi-circle $|z| = 2$ from $-2i$ to $2i$ in the right half-plane.

   (b) $f(z) = e^{2z}$ and C is the shortest path from 0 to $1 + 2i$.

3. Evaluate $\int_C \cot z\, dz$, where C is the circle $\left|z + \frac{1}{2}\right| = \frac{1}{3}$.