

MA 201 - Tutorial set 3

Note: Throughout the tutorial set, $z = x + iy$.

1. Show that if $a < b$ and γ denotes the circle centered at a , traversed in the counterclockwise direction, and which has b in its exterior, then

$$\int_{\gamma} \frac{1}{(z-a)(z-b)} dz = \frac{2\pi i}{a-b},$$

2. Evaluate

$$\int_{\gamma} \frac{1}{z+1} dz,$$

where γ is any curve in $D = \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$, which joins $-1 + i$ to $1 + 2i$.

3. Show that

(a) the function $f(z) = \operatorname{Log}(z - i)$ is analytic everywhere except on the portion $x \leq 0$ of the line $y = 1$;

(b) the function

$$f(z) = \frac{\operatorname{Log}(z+4)}{z^2+i}$$

is analytic everywhere except at the points $\pm \frac{(1-i)}{\sqrt{2}}$ and on the portion $x \leq -4$ of the real axis.

4. Find the principal branch cut of

(a) $\sqrt{1-z^2}$.

(b) $\log(z^2-1)$.

5. Evaluate $\int_C |z| dz$, where C is the circle $|z-1| = 1$ described in the counterclockwise direction.

6. Compute

$$\int_{|z|=1} |z-1| \cdot |dz|.$$