

MA 509: Tutorial 7 (2020)

1. Prove that convergence of $\{s_n\}$ implies convergence of $\{|s_n|\}$. Is the converse true?

2. Calculate $\lim_{n \rightarrow \infty} (\sqrt{n^2 + n} - n)$.

3. Let $\{s_n\}$ be a real-valued sequence. Prove that $\lim_{n \rightarrow \infty} s_n = s$ iff $\limsup_{n \rightarrow \infty} s_n = \liminf_{n \rightarrow \infty} s_n = s$.

4. For any two real sequences $\{a_n\}$ and $\{b_n\}$, prove that

$$\limsup_{n \rightarrow \infty} (a_n + b_n) \leq \limsup_{n \rightarrow \infty} a_n + \limsup_{n \rightarrow \infty} b_n$$

provided the sum on the right is not of the form $\infty - \infty$.

5. Find the upper and lower limits of the sequence $\{s_n\}$ defined by

$$s_1 = 0; \quad s_{2m} = \frac{1}{2}s_{2m-1}; \quad s_{2m+1} = \frac{1}{2} + s_{2m}.$$