MA 509: Tutorial 9 (2020)

1. Let E be a non-compact and bounded subset of $\mathbb{R}$. In the class, we have proved that if x_0 is a limit point of E which is not a point of E, then the function $f(x) = \dfrac{1}{x - x_0}$, where $x \in E$, is continuous at every point in E. Now show that f is not uniformly continuous on E.

2. If f is a continuous mapping of a metric space X into a metric space Y, prove that $f(\overline{E}) \subset \overline{f(E)}$ for every set $E \subset X$. Show, by an example, that $f(\overline{E})$ can be a proper set of $\overline{f(E)}$.

3. Let f be a continuous real function on a metric space X. Let $Z(f)$, called the *zero set of* f be the set of all $p \in X$ at which $f(p) = 0$. Prove that $Z(f)$ is closed.

4. Let $I = [0, 1]$ be the closed unit interval. Suppose f is a continuous mapping of I into I. Prove that $f(x) = x$ for at least one $x \in I$.

5. Call a mapping of X into Y *open* if $f(V)$ is an open set in Y whenever V is an open set in X. Prove that every continuous open mapping of $\mathbb{R}$ into $\mathbb{R}$ is monotonic.