MA 631: Homework 2 (Due September 22)

(Note: Justify all the relevant steps.)

1. We proved in class that

$${}_2F_1(a,b;a+b-c+1;1-z) = P \cdot {}_2F_1(a,b;c;z) + Qz^{1-c}{}_2F_1(a-c+1,b-c+1;2-c;z),$$

where

$$P = \frac{\Gamma(a+b-c+1)\Gamma(1-c)}{\Gamma(a-c+1)\Gamma(b-c+1)}.$$

Prove that

$$Q = \frac{\Gamma(c-1)\Gamma(a+b+1-c)}{\Gamma(a)\Gamma(b)}.$$

2. Derive the Chu-Vandermonde identity by equating the coefficients of x^n on each side of

$$(1-x)^{-a}(1-x)^{-b} = (1-x)^{-(a+b)}.$$

3. Kummer's confluent hypergeometric function is defined by

$${}_1F_1(a;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n} \frac{z^n}{n!}.$$

This series converges absolutely for all $z \in \mathbb{C}$.

Prove Kummer's relation

$${}_1F_1(a;c;z) = e^z {}_1F_1(c-a;c;-z).$$