

MA 631: Homework 3 (Due November 8)

(Note: Justify all the relevant steps.)

1. Prove that the differential equation satisfied by the Laguerre polynomial $L_n^\alpha(x)$ is

$$xy'' + (\alpha + 1 - x)y' + ny = 0.$$

2. Prove that the differential equation satisfied by the Hermite polynomial $H_n(x)$ is

$$y'' - 2xy' + 2ny = 0.$$

3. Let $T_n(x)$ be the Chebyshev polynomial of the first kind of degree n . Show that for $0 \leq \theta \leq \pi$,

$$T_n(\cos \theta) = \cos(n\theta), \text{ or in other words } T_n(x) = \cos(n \cos^{-1}(x)).$$

(Hint: Work with the differential equation satisfied by the Chebyshev polynomial.)