MA 631: Special Functions - Tutorial 1

1. For $m \geq 1$, prove Raabe's multiplication formula for Bernoulli polynomials:

$$\frac{1}{m}\sum_{k=0}^{m-1} B_n\left(x+\frac{k}{m}\right) = m^{-n}B_n(mx).$$

2. Prove the following formulas for $n \in \mathbb{N}$:

$$\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m^{2n}} = \frac{(-1)^{n+1}(2\pi)^{2n}(1-2^{1-2n})B_{2n}}{2(2n)!},$$

$$\sum_{m=0}^{\infty} \frac{1}{(2m+1)^{2n}} = \frac{(-1)^{n+1}(2\pi)^{2n}(1-2^{-2n})B_{2n}}{2(2n)!}.$$

3. Using Euler's summation formula

$$\sum_{i=1}^{n} f(i) = \int_0^n f(x)\,dx + \frac{1}{2}(f(n)-f(0)) + \int_0^n \tilde{B}_1(x)f'(x)\,dx,$$

prove that the limit

$$\gamma = \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n} - \ln n\right)$$

exists. The latter is called Euler's constant.