MA 631: Special Functions - Tutorial 2

1. Using Watson's lemma, prove that the asymptotic expansion of the exponential integral is given by

$$x\int_x^{\infty} t^{-1}e^{x-t}\,dt \sim \sum_{n=0}^{\infty}\frac{(-1)^n n!}{x^n}$$

as $x \to \infty$.

2. Prove the following following variant of the reflection formula:

$$\Gamma\left(\frac{1}{2}-z\right)\Gamma\left(\frac{1}{2}+z\right)=\frac{\pi}{\cos(\pi z)} \qquad (z-\tfrac{1}{2}\notin\mathbb{Z}).$$

3. Show that for $\mathrm{Re}(m)>0$, $\mathrm{Re}(n)>0$, $\mathrm{Re}(b)>0$ and $\mathrm{Re}(c)>0$,

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(b+cx)^{m+n}}\,dx=\frac{B(m,n)}{b^n(b+c)^m},$$

where $B(m,n)$ is Euler's beta integral.