MA 631: Special Functions (2022) - Tutorial 3

1. An integral of the form $\int_0^\infty t^{z-1} f(t)\, dt$ is called the Mellin transform of f. Show that for $0 < \mathrm{Re}(z) < 1$,

$$\int_0^\infty t^{z-1} \cos(t)\, dt = \Gamma(z) \cos\left(\frac{\pi z}{2}\right),$$
$$\int_0^\infty t^{z-1} \sin(t)\, dt = \Gamma(z) \sin\left(\frac{\pi z}{2}\right).$$

2. For $\mathrm{Re}(m) > 0$ and $\mathrm{Re}(n) > 0$, prove that

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(b+cx)^{m+n}}\, dx = \frac{B(m,n)}{b^n (b+c)^m},$$

where $B(m,n)$ is the Euler beta integral.