Solutions to remaining problems in the tutorials

Tut. 3 prob. 4 (Lexicographic ordering)

Does this ordered set have the least upper bound property?

Ans. No. (Counter-example: Let $A = \{(0, y) : y \in \mathbb{R}\}$.
Then $A \neq \phi$. Let $x > 0$. Then each $(x, z) \in \mathbb{R}^2$ is an upper bound of A. However, A does not have a least upper bound.

Tutorial 4 Prob. 4

Ans. • Yes, if E is an open set in $\mathbb{R}^2$.
• No, if E closed.
Eg: Let E be a finite set in $\mathbb{R}^2$.
Then $E' = \phi$.

Tutorial 5 Prob. 4

X is an infinite set. For $p, q \in X$,

$$d(p, q) = \begin{cases} 1, & (p \neq q) \\ 0, & (p = q) \end{cases}$$

• Easy to check that this is a metric.
• Claim: Every subset of this metric space X is open.

Proof: Let E be a singleton set, say,
$E = \{p\} \subset X$ (metric space)

Choose a nbhd $N_r(p)$ for $r<1$. Then

$N_r(p) = \{p\} \subset \{p\} = E$.

$\Rightarrow$ Every singleton set is open. — 1

Since union of any number of open sets is open, every subset of X is open.

Since a set is closed iff its complement is open, every subset of X is closed as well.